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# Perturbative versus nonperturbative effects at medium energies in the $\gamma\gamma \to \pi^+\pi^-$ reaction

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**Abstract.** The interplay of pQCD, pion exchange and FSI effects is studied for the $\gamma\gamma \to \pi^+\pi^-$ reaction in the region of $2\,\mathrm{GeV} < W_{\gamma\gamma} < 6\,\mathrm{GeV}$. We find strong interference effects between pQCD and soft-pion-exchange amplitudes up to $W_{\gamma\gamma} \sim 4\,\mathrm{GeV}$. We discuss to which extend the conventional hadronic FSI effects cloud the pQCD effects. We study multipole soft- and hard-scattering effects as well as the coupling between final-state hadronic channels.

**PACS.** 12.38.Bx Perturbative calculations – 11.80.La Multiple scattering

## 1 Introduction

It was predicted long ago [1] that at large $t$ and large $u$ the angular distribution of pions from the reaction $\gamma\gamma \to \pi^+\pi^-$ should be described by means of perturbative QCD due to the exchange of $t$-channel ($u$-channel) quarks. This reaction is commonly considered as a gold-plated reaction for pQCD effects to be observed. There is no common consensus how large $t$ (or $u$) should be so that the pQCD behaviour of the angular distributions could be observed.

In the leading-twist perturbative treatment, the transition amplitude factorizes into a hard-scattering amplitude and a pion distribution amplitude. The pion distribution amplitude was found to be strongly constrained by the photon-pion transition form factor [2]. For the "realistic" distribution amplitude the pQCD contribution to the reaction $\gamma\gamma \to \pi^+\pi^-$ [3] is well below the experimental data above the resonance region ($2\,\mathrm{GeV} < W < 3.5\,\mathrm{GeV}$) [4, 5]. Recent analyses of the DELPHI and ALEPH Collaborations at LEP2 [6,7] extend the energy range.

Recently we have shown [8] that in the pion-pion elastic scattering at $W > 2\,\mathrm{GeV}$ the multipole soft rescatterings play an important role up to very large momenta transferred. Inspired by the result of [8], we have discussed the role of the final-state interaction (FSI) effects in $\gamma\gamma \to \pi^+\pi^-$ [9].

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## 2 A sketch of calculation details and main results

According to the rules proposed in [10], at large energies and large center-of-mass angles the amplitude for the reaction $\gamma\gamma \to \pi^+\pi^-$ can be factorized into a perturbatively calculable hard-scattering amplitude and a nonperturbative distribution amplitude of finding a valence quark in each pion:

$$M_{\lambda_1\lambda_2}(W,\theta) = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \times \Phi_\pi^*(x_1,\tilde{Q}_1) T^H_{\lambda_1\lambda_2}(x_1,x_2,W,\theta) \Phi_\pi^*(x_2,\tilde{Q}_2). \tag{1}$$

The indices $\lambda_1$ and $\lambda_2$ are photon helicities. In general, the distribution amplitudes $\Phi_\pi$ undergo a slow logarithmic QCD evolution, depending on scales $\tilde{Q}_1$ and $\tilde{Q}_2$. Because in the present analysis we concentrate mainly on pion exchange and FSI effects, we shall limit the approximation for $T^H$ to the leading order (LO) in $\alpha_s$ only. The distribution amplitudes cannot be calculated from first principles. Based on phenomenology of a few reactions, it seems at present that the distribution amplitude is rather close to the asymptotic one:

$$\Phi_\pi(x) = \sqrt{3} f_\pi x(1-x), \tag{2}$$

where $f_\pi$ is the pion decay constant.

In our calculation we followed [11] rather than [1] and used running coupling constant with simple analytic models for freezing [9].

The pQCD amplitude (1) contains singularities at $\theta = 0$ and $\theta = \pi$ (see [1]) which are artifacts of the collinear approximation. Furthermore the small-angle scattering is probably of soft nonperturbative character due to meson or reggeon exchanges which are explicitly included in the present approach. Therefore, in order to avoid double counting and make the use of the pQCD amplitudes in the multiple-scattering series possible, we smoothly cut off the small $t$ and small $u$ regions in the perturbative amplitude [9].

The QED pion-exchange Born amplitude for the reaction $\gamma\gamma \to \pi^+\pi^-$ with point-like particles has been known for long time [12]. An interesting problem is how to generalize the QED amplitude for real finite-size pions. We follow the idea of Poppe [13] and correct the QED amplitude by an overall $t$ and $u$ dependent form factor

$$A_{fs}(t,u,s) = \Omega(t,u,s) \cdot A_{\mathrm{QED}}(t,u,s). \tag{3}$$

This form by construction guarantees crossing symmetry. At sufficiently high energies the following simple Ansatz fulfils this requirement

$$\Omega(t,u,s) = \frac{F^2(t) + F^2(u)}{1 + F^2(-s)}\,, \tag{4}$$

where $F$ is a standard vertex function such that $F(0) \approx 1$ and $F(t) \to 0$ when $t \to \infty$. In the limit of large $s$ the Ansatz (4) generates standard vertex form factors [9]. The finite-size pion exchange leads to a cross-section which decreases with energy faster than the pQCD cross-section. An interesting question is: at what energy the pion exchange can be neglected.

The size of the interference effects can be estimated by comparing the incoherent sum of the pQCD process and the soft pion exchange $\sigma_{\mathrm{inc}} \propto |A_{\mathrm{pQCD}}| + |A_\pi|^2$ and the coherent sum $\sigma_{\mathrm{coh}} \propto |A_{\mathrm{pQCD}} + A_\pi|^2$. It was found [9] that the interference effects are generally large and positive. The coherent sum $|\mathrm{pQCD} + \pi|^2$ depends only weakly on the functional form of the form factor used. The interference of the pQCD and pion exchange amplitudes improves considerably the agreement with the recent DELPHI [6] and ALEPH [7] data. The constructive interference can be observed over the whole angular range [9].

The $f_2$-resonance is known to be strongly populated in the $\gamma\gamma$ collisions (see for instance [14,15]). It is rather broad with $\Gamma \sim 0.2\,\mathrm{GeV}$. The leading-twist pQCD contribution drops strongly with energy. Therefore one should look at the interplay of the high energy flank of $f_2$ with the strongly decreasing continuum. Then all kinds of energy dependence of kinematical and dynamical origin must be included. In the relativistic approach, the total cross-section for the resonance contribution reads

$$\sigma_{\gamma\gamma\to\pi\pi}(W) = 8\pi(2J+1)\left(\frac{M_R}{W}\right)^2 \times \frac{\Gamma_{\gamma\gamma}\Gamma_{\mathrm{tot}}(W) Br\big(f_2 \longrightarrow \pi^+\pi^-\big)}{\big(W^2 - M_R^2\big)^2 + M_R^2\Gamma_{\mathrm{tot}}^2}\,. \tag{5}$$

**Table 1.** Brief summary of double-scattering processes considered in this paper for $\gamma\gamma \to \pi^+\pi^-$.

| Number | First step | Intermediate channel | Second step |
|---|---|---|---|
| 1 | soft $\pi$ exch. | $\pi^+\pi^-$ | $IP + f_2 + \rho$ exch. |
| 2 | pQCD | $\pi^+\pi^-$ | $IP + f_2 + \rho$ exch. |
| 3 | soft VDM | $\rho^0\rho^0$ | $\pi$ exch. |
| 4 | pQCD | $\rho^+(0)\rho^-(0)$ | $\pi$ exch. |

One usually parametrizes $\Gamma_{\mathrm{tot}}$ as

$$\Gamma_{\mathrm{tot}}(W) = \Gamma_{\mathrm{tot}}^0\left(\frac{p}{p_0}\right)^{2l+1}\mathcal{F}_{\mathrm{dyn}}(p), \tag{6}$$

where $\mathcal{F}_{\mathrm{dyn}}(p)$ is a function of dynamical origin usually obtained in a simple nuclear-physics–inspired model of resonances.

It was recently demonstrated that for elastic pion-pion scattering, soft FSI effects lead to a damping of the cross-section at small angles and a considerable enhancement in the region of intermediate angles where they compete with the two-gluon exchange amplitude. In the following we shall discuss only some selected FSI effects for the reaction under consideration. In table 1 we list double-scattering terms considered in the present analysis.

In the language of multiple scattering at high energies [16] the amplitude for the reaction $\gamma\gamma \to \pi^+\pi^-$ can be written as an infinite series of the type

$$\begin{aligned} A_{\gamma\gamma\to\pi^+\pi^-}(s,t,u) &= \sum_\alpha A^{(\alpha)}_{\gamma\gamma\to\pi^+\pi^-}(s,t,u) \\ &+ \sum_{ij}\sum_{\alpha,\beta}\frac{i}{32\pi^2 s}\int \mathrm{d}^2\boldsymbol{k}_1\mathrm{d}^2\boldsymbol{k}_2\delta^2(\boldsymbol{k}-\boldsymbol{k}_1-\boldsymbol{k}_2) \\ &+ A^{(\alpha)}_{\gamma\gamma\to ij}(s,\boldsymbol{k}_1)A^{(\beta)}_{ij\to\pi^+\pi^-}(s,\boldsymbol{k}_2) \\ &+ (\cdots). \end{aligned} \tag{7}$$

Here Greek indices label type of exchange, while Latin indices $ij$, etc. label two-body intermediate states.

The first component in (7) corresponds to single-scattering terms. In the following we shall include only $\alpha = \pi$ (pion exchange) or $\alpha = 2q$ (pQCD quark exchange) single-exchange amplitudes. We include only $ij = \pi^+\pi^-$ and $ij = \rho\rho$ intermediate states. The parameters of pion-pion interaction are taken from ref. [8].

In fig. 1 we present the result corresponding to the coherent sum of all processes discussed. For comparison we show the pQCD result. For $W > 2\,\mathrm{GeV}$ the final result is practically independent of the phase of the resonance contribution with respect to the other contributions. As can be seen by comparison of the solid and dashed line the inclusion of the processes considered in the present paper leads to a considerable improvement with respect to the pQCD calculation. The final result describes the ALEPH [7] and DELPHI [6] data surprisingly well.

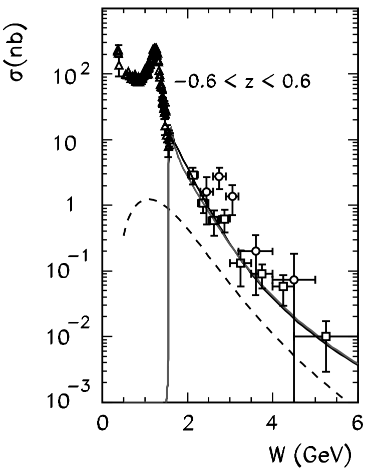


**Fig. 1.** The result corresponding to the coherent sum of all processes (solid line) *versus* pQCD result (dashed line). The vertical solid line shows the expected lower limit of the range of applicability of the present multipole-scattering approach.

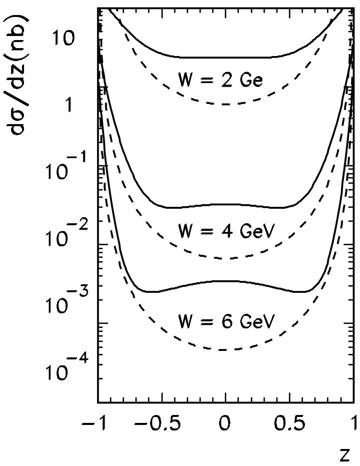


**Fig. 2.** The result corresponding to the coherent sum of all processes (solid line) *versus* pQCD result (dashed line).

To complete our results we present in fig. 2 the corresponding angular distributions. A large enhancement with respect to pQCD at intermediate angles ($z \sim 0$) is clearly visible. We predict an almost flat $\mathrm{d}\sigma/\mathrm{d}z$ in the experimentally measured region of $-0.6 < z < 0.6$. This is due to the competition of pion-exchange, pQCD, the tail of the $f_2$-resonance and multiple scattering effects.

## 3 Conclusions

Recent LEP2 results for the reaction $\gamma\gamma \to \pi^+\pi^-$, when combined with the present understanding of the pion distribution amplitudes, clearly demonstrate a failure of the leading-twist pQCD in explaining the data.

It was found that the soft-pion exchange, which is known to be the dominant mechanism below the $f_2$-resonance, remains important also above the resonance. Because the pQCD contribution strongly decreases with the photon-photon energy the high-energy flank of the broad $f_2$-resonance gives a contribution of comparable size to the pQCD continuum in a rather broad energy range above the resonance.

Using phenomenological, yet realistic, pion-pion interaction found in a recent analysis, we have estimated the contribution of the FSI processes. We have found that pion-pion FSI as well as the coupling with the $\rho\rho$ channel modify the total amplitude at the level of 20%.

In the light of our analysis, we conclude that the reaction $\gamma\gamma \to \pi^+\pi^-$ is not the best choice to identify the pQCD predictions.

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